Calculus AB

5-1

The Natural Logarithm: Differentiation

Algebraic Definition of Logarithm - the answer to a logarithm is an exponent.

Given: $x^b = y \Rightarrow \log_x y = b$

Definition of the Number e - $e = 2.718281828...$

Euler's number

Algebraic Definition of the Natural Logarithm -

$\ln x = \log_e x$

Definition of the Natural Logarithm Function -

$$\ln(x) = \int_1^x \frac{1}{t}\,dt$$

Laws of Exponents	Laws of Logarithms -
1) $x^m \cdot x^n = x^{m+n}$ When multiplying, add powers.	When multiplying 1) $\log_b MN = \log_b M + \log_b N$ Add powers (answer to a logarithm is a ...)
2) $\frac{x^m}{x^n} = x^{m-n}$ When dividing, subtract powers.	When dividing 2) $\log_b \frac{M}{N} = \log_b M - \log_b N$ subtract powers.
3) $(x^m)^n = x^{mn}$ Power of a power, multiply exponents	3) $\underbrace{\log_b M^n}_{\text{Power of a power}} = n\underbrace{\log_b M}_{\text{multiply exponents (answer to a logarithm is a ...)}}$

REMEMBER: The answer to a logarithm is a power.

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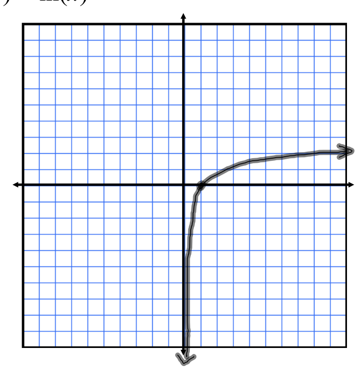
Function Properties: $f(x) = \ln(x)$

Domain: $(0, \infty)$

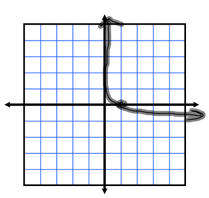
Range: $\mathbb{R}$

Asymptotes: $x = 0$



Sketch the graph of the function and state its domain. (pg 331)

12) $f(x) = -2\ln(x)$



Use the properties of logarithms to expand the logarithmic expression.

22) $\ln\sqrt{x^5} = \ln x^{\frac{5}{2}} = \frac{5}{2}\ln x$

24) $\ln(xyz) = \ln x + \ln y + \ln z$

*) $\ln\sqrt[3]{a^2+1} = \ln(a^2+1)^{\frac{1}{3}}$

$= \frac{1}{3}\ln(a^2+1)$

Write the expression as a logarithm of a single quantity.

30) $\ln x + 2\ln y - 4\ln z$

$$\ln \frac{xy^2}{z^4}$$

34) $2[\ln x - \ln(x+1) - \ln(x-1)]$

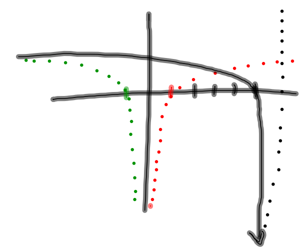
$$\ln\left[\frac{x}{(x+1)(x-1)}\right]^2$$

Find the limit.

$-x+6$

40) $\lim_{x \to 6^-} \ln(6-x) = -\infty$

Note: from left.



$aF(bx-c)+d$

$-\frac{c}{b}$

Use function transformations to visualize graph.

Derivative of the Natural Logarithm Function.

$$\frac{d}{dx}\ln x = \frac{1}{x}, \; x > 0$$

Find the derivative of the function.

50) $h(x) = \ln(2x^2 + 1)$

$$= \frac{4x}{2x^2+1}$$

56) $y = \ln \frac{2x}{x+3}$

$$y = \ln 2x - \ln(x+3)$$

$$\frac{dy}{dx} = \frac{2}{2x} - \frac{1}{x+3}$$

$$= \frac{1}{x} - \frac{1}{x+3}$$

$$\frac{x+3-x}{x(x+3)} = \frac{3}{x^2+3x}$$

70) $y = \ln|\csc x|$

$$\frac{-\csc x \cot x}{\csc x} = -\cot x$$

Assignment

Day 1

pg 331

7-17 odd

21-35 odd

39-75 odd

Find an equation of the tangent line to the graph of f at the indicated point.

78) $f(x) = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right)$, $(0, 4)$

$$F'(x) = -2x - \frac{\frac{1}{2}}{\frac{1}{2}x+1} = -2x - \frac{1}{x+2}$$

$$m = F'(0) = -2(0) - \frac{\frac{1}{2}}{\frac{1}{2}(0)+1} = \boxed{-\frac{1}{2}}$$

$$y = -\frac{1}{2}x + b$$

$$4 = -\frac{1}{2}(0) + b$$

$$4 = b$$

$$\boxed{y = -\frac{1}{2}x + 4}$$

Show that the function is a solution of the differential equation.

90) $x \ln x - 4x = y$

$$y' = \overbrace{\left[1 \cdot \ln x + x\left(\frac{1}{x}\right)\right]}^{\text{product rule}} - 4$$

$$= \ln x - 3$$

$$x + y - xy' = 0$$

$$x + (x\ln x - 4x) - x(\ln x - 3) = 0$$

$$x + \cancel{x\ln x} - 4x - \cancel{x\ln x} + 3x = 0$$

$$0 = 0$$

Locate any relative extrema and inflection points.

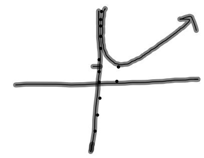
92) $y = x - \ln x$ —Domain $(0, \infty)$

$\frac{dy}{dx} = 1 - \frac{1}{x}$ c.p. $0 = 1 - \frac{1}{x}$, $x = 1$ } undefined at $x=0$, but not in domain

$\frac{d^2y}{dx^2} = \frac{1}{x^2}$ $\Big|_{x=1} = \frac{1}{(1)^2} = 1 \Rightarrow$ min at $(1,1)$

$0 = \frac{1}{x^2}$ no inflection points.

$\varnothing$



Assignment

Day 2

83 - 95 odd

Continue with 5-2, time permitting.